Section 3.8

Newton's Method for Approximating the Zeros of a Function

Let $f(c) = 0$, where f is differentiable on an open interval containing c. Then, to approximate c, use these steps.

1. Make an initial estimate x_1 that is close to c. (A graph is helpful.)
2. Determine a new approximation
$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}.$$
3. When $|x_n - x_{n+1}|$ is within the desired accuracy, let x_{n+1} serve as the final approximation. Otherwise, return to Step 2 and calculate a new approximation.

1) Calculate three iterations of Newton's Method to approximate a zero of $f(x) = x^2 - 3$. Use $x_1 = 2$ as the initial guess. Fill in the table below to help you.

n	x_n	$f(x_n)$	$f'(x_n)$	$\frac{f(x_n)}{f'(x_n)}$	$x_n - \frac{f(x_n)}{f'(x_n)}$
1					
2					
3					

2) Use Newton's Method to approximate the zeros of $f(x) = x^3 - 2x^2 + 4x + 2$. Continue the iterations until two successive approximations differ by less than 0.0001. As in the example, graph $f(x)$ on your calculator to get a good first approximation of the zero. Fill in the table below to help.

n	x_n	$f(x_n)$	$f'(x_n)$	$\frac{f(x_n)}{f'(x_n)}$	$x_n - \frac{f(x_n)}{f'(x_n)}$
1					
2					
3					
4					

Homework for 3.8: #1, 7, 22